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Asymptotic behaviour of fundamental solutions of elliptic operators with order higher than two

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Abstract. One studies asymptotic behaviour at infinity of fundamental solutions to elliptic PDE order higher than two. The main tool to be used is estimates for oscillatory integrals obtained by A.N. Varchenko.

Key words: partial differential operator of elliptic type, singularity theory, asymptotic analysis.

Introduction

In this article, we shall investigate the following problem. Let $F(D)$ be a homogeneous elliptic differential operator of order $2m$ $(m \geq 2)$ with constant coefficients on $\mathbb{R}^N$:

$$F(D) = \sum_{|\alpha|=2m} F_\alpha D^\alpha, \tag{0.1}$$

where $F(\xi) > 0$ for $\xi \in \mathbb{R}^N \setminus 0$, $F_\beta = 1$ for $\beta = (0, 0, \ldots 0, \overset{i}{\check{2m}}, 0, \ldots 0)$, $1 \leq i \leq N$.

Let $E(x)$ be the fundamental solution to the operator $F(D) - I$ that is bounded as x tends to infinity,

$$F(D)E(x) - E(x) = \delta(x). \tag{0.2}$$

The problem is to know asymptotic behaviour of $E(x)$ as x tends to infinity, starting from the coefficients F_α, $\alpha \in \mathbb{N}^N$. In the case when the energy surface $S = \{F(\xi) = 1\}$ has no point where its total curvature vanishes, the answer to this problem has already been gotten by B.R. Vainberg [Vai], as we shall review in §1. Therefore we are interested in the cases where the total curvature may vanish at some points on the energy surface.

In §1 we remember the fundamental tools that help us to reduce the study on $E(x)$ to that of oscillatory integrals with degenerate phases.

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