Hokkaido Mathematical Journal Vol. 29 (2000) p. 329–339

L^p-L^q asymptotic behaviors of the solutions to the perturbed Schrödinger equations

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(Received April 5, 1999; Revised September 24, 1999)

Abstract. We consider the asymptotic behaviors of a solution to the Schrödinger equation as t goes to $\pm\infty$. We present the sharp asymptotics in $L^\infty(\mathbf{R}^m)$. In particular, the low energy part of the perturbed dynamics is dominant to the L^∞ scattering and an explicit asymptotic form is shown in the uniform convergence topology.

Our approach to prove the results is based on the application of two facts, i.e., the local energy decay of $e^{-itH}P_{ac}(H)$ due to Jensen-Kato [4] and the L^p-boundedness properties of wave operators due to Yajima [10].

Key words: scattering theory, wave operators, L^∞, zero energy resonance, Schrödinger group.

1. Introduction

In this paper, we consider the asymptotic behaviors of the solution to the linear Schrödinger equation of the following type:

$$\begin{cases} i\partial_t u = Hu, \\ u|_{t=0} = \phi, \end{cases} \tag{1.1}$$

where u is a complex valued function of $(t,x) \in \mathbf{R} \times \mathbf{R}^m$ $(m \geq 3)$. H is the Hamiltonian of the form $H = H_0 + V$, where $H_0 = -\Delta$ and V is a short range scalar potential.

If the potential V is rapidly decreasing as $|x| \to \infty$, a particle governed by the above dynamics is expected to be asymptotically free for large time t. In view of the quantum mechanics, this means that the quantum state approaches to the free state. Mathematically, the solution $u(t,x)$ of (1.1) tends to the solution $v(t,x)$ of the free equation $i\partial_t v = H_0 v$. More precisely, by introducing the wave operators $W_\pm \equiv s\text{-}\lim_{t\to\pm\infty} e^{itH}e^{-itH_0}$ in $L^2(\mathbf{R}^m)$, it is well-known that, if ϕ belongs to the absolutely continuous part of H, then $\|e^{-itH}\phi - e^{-itH_0}W_\pm^*\phi\|_{L^2} \to 0$ as $t \to \pm\infty$, where $W_\pm^*$ are the adjoint operators of $W_\pm$. This scattering result can be extended into the general

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1991 Mathematics Subject Classification : 47A40.