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Mod p cohomology algebras of finite groups with extraspecial Sylow p-subgroups

(Dedicated to Professor Yukio Tsushima on his sixtieth birthday)

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Abstract. We analyze mod p cohomology algebras of finite groups with extraspecial Sylow p-subgroups by applying the theory of relative projectivity of modules, which is of fundamental importance in the modular representation theory of finite groups, to the cohomology theory. Especially we shall calculate the mod p cohomology algebras of the general linear group $\mathrm{GL}(3, \mathbf{F}_p)$.

Key words: cohomology of finite groups, relative projectivity of modules, Carlson modules, Green correspondence.

1. Introduction

Let p be a prime greater than three. In this paper we consider cohomology algebras of finite groups with extraspecial Sylow p-subgroup

$$P = \langle a, b \mid a^p = b^p = [a, b]^p = 1, [[a, b], a] = [[a, b], b] = 1 \rangle$$

of order p^3 and exponent p with coefficients in fields of characteristic p.

Integral cohomology rings of these finite groups have been investigated by some people. Among them we should mention D. J. Green [6] and Tezuka-Yagita [11]. Green's work would be the first one dealing with such finite groups and contains a useful proposition that can be applied to modular case. Tezuka and Yagita's work is a comprehensive one considering finite simple groups with P as Sylow p-subgroups and gave universally stable classes. Some of these results and methods are valid for modular cases. The present paper is partly inspired by their works.

We should also mention Milgram-Tezuka [8]. There they calculated the mod 3 cohomology algebra of the Mathieu group M_{12}, whose Sylow 3-subgroup is extraspecial of order 27 and exponent 3; and they showed

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