

## Oscillations of delay difference equations

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**Abstract.** We obtain some new sufficient conditions for oscillations of all solutions of the delay difference equation

$$y_{n+1} - y_n + p_n y_{n-k} = 0, \quad n = 0, 1, 2, \dots$$

where  $\{p_n\}$  is a sequence of nonnegative numbers and  $k$  is a positive integer. Our theorems improve several previous well-known results. Some examples are given to demonstrate the advantage of our results.

*Key words:* oscillation, eventually positive solution, difference equation.

### 1. Introduction

In the recent papers [1–12], the oscillation of all solutions of the delay difference equation

$$y_{n+1} - y_n + p_n y_{n-k} = 0, \quad n = 0, 1, 2, \dots \quad (1)$$

has been investigated, where  $\{p_n\}$  is a sequence of nonnegative numbers and  $k$  is a positive integer.

A solution  $\{y_n\}$  of Eq.(1) is said to be oscillatory if the terms  $y_n$  of the sequence are not eventually positive or eventually negative. Otherwise, the solution is called nonoscillatory.

In [1], Erbe and Zhang first proved that all solutions of (1) oscillate if

$$\liminf_{n \rightarrow \infty} p_n > \frac{k^k}{(k+1)^{k+1}}, \quad (2)$$

or

$$\Lambda = \limsup_{n \rightarrow \infty} \sum_{i=n-k}^n p_i > 1. \quad (3)$$

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