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Crossed products of UHF algebras by some amenable groups

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Abstract. Let A be a UHF C^*-algebra. It is shown that for every homomorphism $\alpha : \mathbb{Z}^n \to \mathrm{Aut}(A)$ there exists an AF embedding $\rho : A \rtimes_\alpha \mathbb{Z}^n \hookrightarrow B$ such that $\rho_* : K_0(A \rtimes_\alpha \mathbb{Z}^n) \to K_0(B)$ is also injective.

Using Green's imprimitivity theorem it will follow that if A is UHF and $\alpha : G \to \mathrm{Aut}(A)$ is a homomorphism then $A \rtimes_\alpha G$ is always quasidiagonal for a large class of amenable groups including all extensions of discrete abelian groups by compact (not necessarily discrete or abelian) groups.

Key words: quasidiagonality, crossed products, K-theory, amenable groups.

1. Introduction

Quasidiagonal C^*-algebras are those which enjoy a certain local finite dimensional approximation property (cf. [Vo2]). In light of some remarkable recent results (cf. [DE], [Li]), it appears that quasidiagonality will play an increasingly important role in Elliott's classification program. A C^*-algebra is called AF embeddable if it is isomorphic to a subalgebra of an AF algebra. Blackadar and Kirchberg asked if the notions of quasidiagonality and AF embeddability agree for nuclear C^*-algebras and there is reason to believe that they do (cf. [BK]). Unfortunately, neither of these notions behave well under taking crossed products (unless the group is compact). Indeed, Voiculescu has asked when $C(X) \rtimes_\varphi \mathbb{Z}^2$ is AF embeddable ([Vo3]) which illustrates how much we have yet to learn in this direction. (Recall that Pimsner characterized the AF embeddability (and quasidiagonality) of $C(X) \rtimes_\varphi \mathbb{Z}$ more than 15 years ago in [Pi].)

In this note we study the AF embeddability and quasidiagonality of crossed products of UHF algebras by certain amenable groups. Our main result is the following.

Theorem 1 *If A is a UHF algebra and $\alpha : \mathbb{Z}^n \to \mathrm{Aut}(A)$ is a homomor-*

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