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Note on MDS codes over the integers modulo p^{m}

Keisuke SHIROMOTO

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Abstract. Recently, a number of papers have been published dealing with codes over finite rings. In this paper, we consider maximum distance separable (MDS) codes over the integers modulo p^{m}, where p is a prime number.

Key words: linear codes over rings, MDS code, module, Singleton bound, Hamming weight, generator matrix.

1. Introduction

In [4], Forney introduced a Singleton bound for codes over any finite alphabet A as follows;

$$d(C)\leq n-k+1,$$

where C is a code of length n over A, $k=\log_{|A|}|C|$ and $d(C)$ is the minimum distance of C and proved several nonexistence results for MDS group codes over finite groups with respect to the above bound, that is, the group codes with $d(C)=n-k+1$. Zain and Rajan [9] also proved that for a group code C over a cyclic group of m elements with generator matrix of the form $(I_{k}\mid M)$, where M is a $k\times(n-k)$ matrix over $\mathbb{Z}_{m}$, C is MDS iff the determinant of every $h\times h$ submatrix, $h=1,2,\ldots,\min\{n-k,k\}$, of M is a unit in $\mathbb{Z}_{m}$. Moreover, Dong, Soh and Gunawan [3] proved a similar matrix characterization of MDS (free) codes with parity check matrices of the form $(-M\mid I_{n-k})$ over modules.

Recently, Shiromoto and Yoshida [8] introduced a Singleton bound for linear codes over $\mathbb{Z}_{k}$ as follows:

Proposition 1 (Shiromoto and Yoshida [8]) *Let C be a linear code of length n over $\mathbb{Z}_{k}$ with the minimum weight $d(C)$. Then,*

$$d(C)\leq n-\mathrm{rank}(C)+1.$$

In the next section, we shall introduce some definitions and notations

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