

Global attractivity of a nonautonomous discrete logistic model

Qinqin ZHANG and Zhan ZHOU

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Abstract. In this paper we consider the nonautonomous discrete logistic model

$$x_{n+1} = x_n \exp[r_n(1 - x_n)], \quad n \in N, \quad (1.1)$$

where $\{r_n\}$ is a sequence of nonnegative numbers. We obtain some sufficient conditions for an arbitrary solution $\{x_n\}$ satisfying the initial condition

$$x_0 = a > 0, \quad (1.2)$$

to converge to 1 as $n \rightarrow \infty$. Under appropriate hypotheses, the necessary and sufficient conditions for any solution of (1.1) with (1.2) tending to 1 as $n \rightarrow \infty$ have also been obtained.

Key words: discrete nonautonomous logistic model, global attractivity.

1. Introduction

Consider the discrete nonautonomous logistic model

$$x_{n+1} = x_n \exp[r_n(1 - x_n)], \quad n \in N, \quad (1.1)$$

where $\{r_n\}$ is a sequence of nonnegative numbers. It is easy to see that, for any given initial condition

$$x_0 = a > 0, \quad (1.2)$$

Eq. (1.1) has an unique solution $\{x_n\}$ which is positive for all $n \in N$ and satisfies (1.2). In [1], it was proved that every solution of (1.1) with (1.2) tends to 1 if $r_n \leq 3/2$ and $\sum_{n=0}^{\infty} r_n = \infty$.

When $r_n \equiv r > 0$, Eq. (1.1) reduces to

$$x_{n+1} = x_n \exp[r(1 - x_n)], \quad n \in N, \quad (1.3)$$