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Existence of δ_m-periodic points for smooth maps of compact manifold*

Grzegorz GRAFF

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Abstract. For a smooth self-map f of a compact manifold M we examine the connection between topological conditions put on M and differentials of a map f at periodic points.

Key words: periodic points, Lefschetz number, cohomological algebra.

1. Introduction

A classical example of the connection between global and local properties of a compact manifold M is Poincaré theorem: $\sum_{x\in C}\operatorname{ind}(T,x)=\chi(M)$, where $\chi(M)$ denotes the Euler characteristic of M, C is the set of critical points of the vector field T, and $\operatorname{ind}(T,x)$ the local index of T.

In 1983 Chow, Mallet-Paret and Yorke ([CMY]) proved that the sequence $\operatorname{ind}(f^n,x_0)$ of isolated fixed point indices of iterated C^1-map f is an integral linear combination of elementary periodic sequences with the periods determined by the spectrum of the derivative $Df(x_0)$ of f at x_0.

Basing on this fact Matsuoka and Shiraki ([MS]) formulated for self-maps of a compact manifold M with finitely many periodic points a global homological condition on M that forces an existence of a periodic point (so called a δ_m-periodic point) which satisfies a certain degeneracy condition.

On the other hand Marzantowicz and Przygodzki ([MP]) expressed a formula for $i_m(f)=\sum_{k|m}\mu(k)I(f^{m/k})$, where $I(f)$ is the fixed point index of f, in terms of periodic points of a compact manifold. If $i_m(f)\neq 0$ then we say that m is an algebraic period of f.

The aim of this paper is to prove the theorem analogous to given in [MS] but formulated in the language of algebraic periods. This approach is more general: we show that both theorems are equivalent for the class of maps with finitely many periodic points, but by a use of algebraic periods it

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