

Mutual exclusiveness among spacelike, timelike, and lightlike leaves in totally geodesic foliations of lightlike complete Lorentzian two-dimensional tori

Ken YOKUMOTO

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Abstract. In this paper we prove an equation for totally geodesic foliations of pseudo-Riemannian manifolds which is originally established in Riemannian case. We prove that if \mathcal{F} is a totally geodesic foliation of a lightlike complete Lorentzian 2-torus T^2 , then \mathcal{F} consists of only one kind of leaves among spacelike, timelike, and lightlike ones. As a corollary we prove that a totally geodesic foliation of a lightlike complete 2-torus has no Reeb components.

Key words: pseudo-Riemannian manifolds, totally geodesic foliations.

1. Introduction

In this section we explain what motivated us to consider totally geodesic foliations of pseudo-Riemannian manifolds.

There are a lot of results about codimension-one totally geodesic foliations of complete Riemannian manifolds as follows. Let \mathcal{F} be a codimension-one totally geodesic foliation of a complete Riemannian manifold (M, g) . The universal covering of M is a product $L \times \mathbf{R}$ and the lift of \mathcal{F} is the product foliation, where L is the universal covering of the leaves of \mathcal{F} (see [BH]). The foliation perpendicular to \mathcal{F} is a Riemannian foliation ([CG]). G. Oshikiri proved that any Killing field with bounded length preserves \mathcal{F} (see [Os1], [Os2]). In [Gh], E. Ghys classified totally geodesic foliations of a closed Riemannian manifold.

On the other hand, we do not know so many results on totally geodesic foliations of pseudo-Riemannian manifolds. Hence we consider those of pseudo-Riemannian manifolds, in particular, those of Lorentzian manifolds. We recall some results about them. In [Z], A. Zeghib constructed codimension-one, lightlike totally geodesic foliations. He made a foliation \mathcal{F} lightlike totally geodesic in two cases: