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Harmonic univalent functions with fixed second coefficient

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Abstract. We define and investigate a family of harmonic univalent functions that have fixed second coefficient. Extreme points, convolution conditions, convex combinations, and distortion bounds are obtained for these functions.

Key words: harmonic, univalent, starlike, convex.

1. Introduction

Denote by $\mathcal{H}$ the family of functions $f = h + \bar{g}$ which are harmonic univalent and sense-preserving in the open unit disk $\Delta = \{z : |z| < 1\}$ where h and g are given by

$$h(z) = z + \sum_{n=2}^{\infty} a_n z^n, \quad g(z) = \sum_{n=1}^{\infty} b_n z^n, \quad |b_1| < 1. \tag{1}$$

Note that the family $\mathcal{H}$ reduces to the class $\mathcal{S}$ of normalized analytic univalent functions whenever the co-analytic part of $f = h + \bar{g}$ is zero, i.e., $g \equiv 0$. Also, note that if $f = h + \bar{g} \in \mathcal{H}$, then $h'(0) = 1 > |g'(0)| = |b_1|$. It, therefore, follows that $(f - \overline{b_1 f})/(1 - |b_1|^2) \in \mathcal{H}$ whenever $f \in \mathcal{H}$. With this in mind, we consider $\mathcal{H}^o \subset \mathcal{H}$ for which $b_1 = f_{\bar{z}}(0) = 0$. Clunie and Sheil-Small [8] observed that both $\mathcal{H}$ and $\mathcal{H}^o$ are normal families. They also found that $\mathcal{H}^o$ is compact, while $\mathcal{H}$ is not.

Let $\mathcal{TH}$ be the class of functions in $\mathcal{H}$ that may be expressed as $f = h + \bar{g}$ where

$$h(z) = z - \sum_{n=2}^{\infty} |a_n| z^n, \quad g(z) = \sum_{n=1}^{\infty} |b_n| z^n, \quad |b_1| < 1. \tag{2}$$

For $0 \leq \alpha < 1$, also let $\mathcal{THS}^*(\alpha)$ and $\mathcal{THK}(\alpha)$ be the subclasses of $\mathcal{TH}$ consisting, respectively, of functions starlike of order α and convex of order α in Δ.

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