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# A note on solvability of factorizable finite groups

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**Abstract.** Using theorems on the classification of finite simple groups, we give an extension of some results on the solvability of factorizable finite groups that are generalizations of a well known theorem due to O. Kegel and H. Wielandt.

*Key words*: factorizable groups, 2-decomposable and 2-nilpotent subgroups.

## 1. Introduction

Groups that can be written as a product $G = HK$ of two of its subgroups $H$ and $K$ have been studied by many authors. Based on Kegel-Wielandt's theorem [6, Satz 4.3, p.674], which states that a finite group is solvable if it is the product of two nilpotent subgroups. Similar problems on factorizable groups have been studied by various authors.

In [3] and [8], factorizable groups $G = HK$ are studied, where $H$ is 2-decomposable and $K$ is nilpotent of odd order. Here a finite group $H$ is called 2-*decomposable* if it is the direct product of a Sylow 2-subgroup, with $\mathbf{O}(H)$ the largest normal subgroup of $H$ of odd order. When $H$ is only a product of $\mathbf{O}(H)$ with a Sylow 2-subgroup, it is called 2-*nilpotent*. In [1] (also see [9]), we attempted to generalize the 2-decomposability of $H$ to 2-nilpotency. However, we did not succeed completely. Imposing a stronger restriction on $K$, we obtain the following result.

*Let $G = HP$ be a group such that $H$ is 2-nilpotent and $P$ is a $p$-group of odd order. Then $G$ is solvable.*

The following is a generalization of the result above, which we obtain by removing the restriction on $K$.

**Theorem 1** *Let $G = HK$ be a finite group such that $H$ is 2-nilpotent and $K$ nilpotent of odd order. Then $G$ is solvable.*

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