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On the connection of coefficient and structural conditions about Fourier series

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Abstract. We extend the validity of some theorems treating the relations of coefficient and structural conditions with respect to Fourier series. The extension means that the conditions given by means of the function x^{β}, $\beta>0$, are replaced by concave or Mulholland-type functions.

Key words: Fourier series, coefficient and structural condition, concave function, Mulholland function, best approximation.

1. Introduction

Let $f(x)$ be a 2π-periodic Lebesgue integrable to the pth power $(p>1)$ function and let

$$\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cos nx+b_n\sin nx)$$

be its Fourier series. Denote

$$\rho_n:=(a_n^2+b_n^2)^{1/2}\quad\text{and}\quad p':=\frac{p}{p-1}.$$

In an old-time paper [4], among others, we proved the following result.

Theorem A *Let $w(x)$ $(x\geq 1)$ be a positive and monotone function with the property $w(2n)\leq Aw(n)$ $(A\geq 2,\ n=1,2,\ldots)$, moreover let $0<\beta\leq p'$.*

(i) *If $p\leq 2$ then*

$$\int_0^1 t^{-2}w\left(\frac{1}{t}\right)\left(\int_0^{2\pi}|f(x+2t)+f(x-2t)-2f(x)|^p dx\right)^{\beta/p}dt<\infty \tag{1.1}$$

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