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Tortile Yang-Baxter operators for crossed group-categories

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Abstract. The notion of a tortile Yang-Baxter operator in a crossed group-category is introduced. It is shown that a tortile Yang-Baxter operator on an object X induces a unique braiding and a twist on the free crossed group-category generated by the objects X and X^{*}.

Key words: tortile Yang-Baxter operator, crossed group-category.

1. Introduction

The category of tangles in 3 dimension has a beautiful algebraic characterization in terms of a universal property. This was initially developed by Yetter [10], Turaev [8], Freyd-Yetter [1] and Joyal-Street [3], and has culminated in the work of Shum [7] asserting that the category of framed tangles $\mathcal{FT}$ is monoidally equivalent to the tortile category freely generated by a single object. Joyal and Street [2] gave another purely algebraic interpretation of this category as the free tensor category containing an object equipped with a tortile Yang-Baxter operator.

Recently, Turaev [9] introduced the notion of a modular crossed group-category, and used it to develop 3-dimensional homotopy quantum field theory (HQFT). He started with defining the notion of a tortile (ribbon) crossed π-category for a group π, and showed that modular crossed π-categories induce invariants of 3-dimensional π-manifolds.

The aim of this paper is to give the Joyal and Street's interpretation for a crossed group-category. To do this, we define a balanced Yang-Baxter operator and a tortile Yang-Baxter operator in a crossed group-category. Then we prove that the free crossed group-category $\mathcal{F}$ generated by a single object equipped with a tortile Yang-Baxter operator admits a unique braiding and a twist. Although our construction owes much to the paper [2], several new aspects appear. First, it turns out that one should define a twist

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