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# A characterization of dense vector fields in $\mathcal{G}^1(M)$ on 3-manifolds

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**Abstract.** Recently Morales-Pacfico-Pujals introduced the new concept of singular hyperbolicity and showed that $C^1$ robust transitive sets of 3-flows are singular hyperbolic sets ([8], [9]). Based on their papers, we shall characterize a dense subset of $\mathcal{G}^1(M)$ with $\dim M = 3$.

*Key words*: $\mathcal{G}^1(M)$, singular hyperbolic set, Axiom A.

## 1. Introduction

The purpose of this paper is to study the space of vector fields known as $\mathcal{G}^1(M)$. Let $M$ be a compact smooth manifold without boundary. We denote by $\chi^1(M)$ the set of $C^1$ vector fields on $M$, endowed with the $C^1$ topology and by $X_t$ ($t \in \mathbb{R}$) the $C^1$ flow on $M$ generated by $X \in \chi^1(M)$. $\Omega(X)$, $per(X)$, $Sing(X)$ are the sets of nonwandering, periodic and singular points of $X$ respectively. Recall that a set $\Lambda \subset M$ is called a hyperbolic set of $X$ if compact, invariant and there exists a continuous splitting $TM/\Lambda = E^s \oplus E^X \oplus E^u$, invariant under the derivative of flow $X_t$, $DX_t$, where $E^s$ and $E^u$ are exponentially contracted and expanded respectively by $DX_t$ and $E^X$ is tangent to $X$. We say that $X \in \chi^1(M)$ satisfies Axiom A if $\Omega(X)$ is a hyperbolic set of $X$ and $\Omega(X) = \overline{Sing(X) \cup per(X)}$ (We denote by $\overline{A}$ the closure of $A$ in $M$). Let $\mathcal{G}^1(M)$ be the interior of the set of vector fields in $\chi^1(M)$ whose critical elements (singularities and periodic orbits) are hyperbolic.

In [3], Hayashi showed that diffeomorphisms in $\mathcal{F}^1(M)$ satisfy Axiom A where $\mathcal{F}^1(M)$ is the diffeomorphism version of $\mathcal{G}^1(M)$ and this naturally give rise to the following question: Do vector fields in $\mathcal{G}^1(M)$ satisfy Axiom A? Unfortunately this does not hold generally and the geometric Lorenz attractor in [2] is well-known as one of the counter examples. Vector field generating this attractor is an element of $\mathcal{G}^1(M)$ but has singularities accu-

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