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Absolute continuity of analytic measures

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Abstract. We give an extension of a result due to Asmar, Montgomery-Smith and Saeki, which is concerned with absolute continuity of analytic measures. We also discuss the relation between the space $N(\sigma)$ and absolute continuity of analytic measures.

Key words: LCA group, measure, Fourier transform, absolute continuity.

1. Introduction

Let G be a LCA group with dual group $\hat{G}$. Let $L^1(G)$ and $M(G)$ be the group algebra and the measure algebra, respectively. Let ψ be a nontrivial continuous homomorphism from $\hat{G}$ into $\mathbb{R}$, and let $\phi : \mathbb{R} \to G$ be the dual homomorphism of ψ. Defining an action of $\mathbb{R}$ on G by $t \cdot x = \phi(t) + x$ ($t \in \mathbb{R}$, $x \in G$), we get a transformation group $(\mathbb{R}, G)$. Let σ be a quasi-invariant, (positive) Radon measure on G, and set $N(\sigma) = \{\mu \in M(G) : \phi(h) * \mu \ll \sigma$ $\forall h \in L^1(\mathbb{R})\}$. Then $N(\sigma)$ is an $L^1(\mathbb{R})$-module and an L-subspace of $M(G)$. In general, we have

$$L^1(\sigma) \subset N(\sigma) \subset M(G).$$

According to choice of G and σ, it may happen that $N(\sigma) = M(G)$ and $L^1(\sigma) \subsetneq N(\sigma) \subsetneq M(G)$ (cf. [6] and [14]). Any analytic measure in $N(\sigma)$ is absolutely continuous with respect to σ (Corollary 2.1 or [14, Corollary 2.1]). We show that $N(\sigma)$ is the largest $L^1(\mathbb{R})$-module, L-subspace of $M(G)$ such that any its analytic measure is necessarily absolutely continuous with respect to σ (Corollary 2.3). Recently, Asmar, Montgomery-Smith and Saeki obtained a new version of Bochner's generalization of the F. and M. Riesz theorem ([3, Theorem 4.5]). We also give another proof of it (Theorem 2.2).

2. Notation and results

Let G be a LCA group with dual group $\hat{G}$. We denote by $\mathfrak{B}(G)$ the σ-algebra of Borel sets in G. For $x \in G$, δ_x denotes the point mass at x. We

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