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# Extension of holomorphic functions through a hypersurface by tangent analytic discs

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**Abstract.** We prove a new criterion of extendibility of holomorphic functions from a domain $\Omega\subset\mathbb{C}^{n}$ by means of analytic discs $A$ tangent to $\partial\Omega$, and verifying $\partial A\subset\bar{\Omega}$ and $\partial A\cap\Omega\neq\emptyset$.

*Key words*: CR functions, analytic discs.

## 1. Introduction

Let $\Omega$ be a domain of $\mathbb{C}^{n}$ with boundary $M$, $A$ an analytic disc *attached* to $\bar{\Omega}$ and not to $M$, that is verifying $\partial A\subset\bar{\Omega}$ but $\partial A\not\subset M$. We assume $A$ to be tangent to $M$ at some boundary point $z^{o}\in\partial A\cap M$, and let $B$ be a ball with center $z^{o}$ which contains $\bar{A}$. Then holomorphic functions $f$ on $\Omega\cap B$ extend holomorphically to a fixed neighborhood of $z^{o}$ (Theorem 2.1 herein). In spite of its appearence, this is in fact a result on propagation of holomorphic extendibility from $\Omega$ along boundaries of discs attached to $M$ at points of tangency (Theorem 2.2 herein). The first result of this type is the HANGES-TREVES Theorem in [6] on propagation along discs which are entirely contained in $M$. Successively, in the paper [9], TUMANOV proved that *defective* discs attached to $M$ are as well *propagators* of extendibility. Note that defective discs are all tangent but they do not exhaust the class of tangent discs. Also, if we assume that $\bar{A}\subset\bar{\Omega}$ but $\bar{A}\not\subset M$ in any neighborhood of $z^{o}$, then our result implies extension of germs at $z^{o}$ of holomorphic functions on $\Omega$. We recall that extendibility of germs (to either side of $M$) is equivalent to *minimality* of $M$ in the sense of TREPREAU and TUMANOV. Hence, if $M$ is not minimal, then one-sided discs tangent to $M$ at $z^{o}$ are in fact entirely contained in $M$ (Proposition 4.1 herein). The method of the present paper consists in the analysis of the properties of superharmonicity of $\log r_{f}^{\nu}$ where $r_{f}^{\nu}$ denotes the radius of convergence of the Taylor series of a holomorphic function $f$ on $\Omega$ in a direction $\nu\in\mathbb{C}^{n}\setminus\{0\}$. Another

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