

Weighted weak type inequalities for maximal commutators of Bochner-Riesz operator

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Abstract. In this paper, we establish endpoint estimates of $L(\log L)$ type for maximal commutators of Bochner-Riesz operators, and the weighted weak type estimates for the commutators are also obtained.

Key words: Bochner-Riesz operator, commutator, weak type inequality, weight, sharp function.

1. Introduction

Let $b \in BMO(R^n)$ and T be a Calderon-Zygmund operator. Consider the commutator defined by

$$[b, T]f(x) = b(x)Tf(x) - T(bf)(x).$$

A classical result of Coifman, Rochberg and Weiss [3] proved that $[b, T]$ is bounded on $L^p(R^n)$ ($1 < p < \infty$). However, it is observed that $[b, T]$ is not, in general, weak type (1,1). In fact, Perez [10] proved that $[b, T]$ satisfies $L(\log L)$ type inequality. The purpose of this paper is to consider a similar problem: how to establish the weak type inequalities for the maximal commutators of Bochner-Riesz operators. Recently, the boundedness of the commutators on $L^p(R^n)$ and Herz-type Hardy spaces are studied in [7], [9], we go on doing this work. We show that the commutators satisfy $L(\log L)$ type inequalities, and the weighted weak type inequalities for the commutators are also obtained. In Section 2 and 3, we will give some concepts and Theorems of this paper, whose proofs will appear in Section 5, and Section 4 contains some Lemmas.

2. Definition

Let us first introduce some concepts.