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Remark on application of distribution function inequality for Toeplitz and Hankel operators

Michiaki HAMADA

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Abstract. In this paper we characterize the compact product of analytic Toeplitz operator and Hankel operator, and the compact commutator of two Hankel operators, by using some distribution function inequalities.

Key words: Toeplitz and Hankel operators, distribution function inequality.

1. Introduction

Let $\mathbb{D}$ be the open unit disk in the complex plane and $\partial\mathbb{D}$ be the unit circle. Let dA denote the normalized Lebesgue measure on $\mathbb{D}$ and $d\sigma$ denote the normalized Lebesgue measure on $\partial\mathbb{D}$. The Lebesgue space L^2 is the space of square integrable functions on $\partial\mathbb{D}$ and the Hardy space H^2 is the closed subspace of L^2 which is spanned by analytic polynomials. For f in L^∞, the space of essentially bounded functions on the unit circle, Toeplitz operator T_f and Hankel operator H_f on Hardy space H^2 is defined by $T_f g = P(fg)$ and $H_f g = J(I-P)(fg)$, where P is the orthogonal projection from L^2 onto H^2 and J is the unitary operator on L^2 defined by $Jg(w) = \overline{w}g(\overline{w})$. It is easily seen that $J^2 = I$, $J(I-P) = PJ$. This definition of Hankel operator may not be standard because many authors call next operator $\mathcal{H}_f$ Hankel operator: $\mathcal{H}_f g = (I-P)(fg)$. Clearly $\mathcal{H}_f$ is bounded transformation of H^2 to $(H^2)^\perp$ and $H_f = J\mathcal{H}_f$. H_f and $\mathcal{H}_f$ have many similar properties. For example matrix representatioins of H_f and $\mathcal{H}_f$ with respect to standard basis of H^2 and $(H^2)^\perp$ are both characterized that the entries on each skew-diagonal direction are the same constant. In this paper we are mainly interested in Hankel operator H_f.

Many authors have studied Toeplitz and Hankel operators with respect to the compact operators, and I think one of the most beautiful results of these operators are Axler-Chang-Sarason-Volberg theorem ([1], [13]). In 1970's they characterized the condition for the compactness of semi-

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