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Another example of an invariant subspace of H^{∞} with index $\mathfrak{c}$

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Abstract. A. Borichev gave an example of an invariant subspace $\mathcal{M}$ of H^{∞} with $\dim \mathcal{M}/z\mathcal{M} = \mathrm{card}[0,1] = \mathfrak{c}$, which is generated by an uncountable family of Blaschke products. In this paper, we construct singular inner functions which generate an invariant subspace $\mathcal{M}$ with $\dim \mathcal{M}/z\mathcal{M} = \mathrm{card}[0,1]$.

Key words: invariant subspace, index, singular inner function.

1. Introduction

Let $L_a^2(D)$ be the Bergman space of all analytic functions on the open unit disc D in the complex plane that satisfy the following condition:

$$\int_D |f(z)|^2 dA(z) < +\infty,$$

where dA is the normalized area measure in D. A closed subspace $\mathcal{M}$ of $L_a^2(D)$ is said to be (z-) invariant if $zf \in \mathcal{M}$ whenever $f \in \mathcal{M}$. Here, z is the coordinate function. The dimension of the quotient space $\mathcal{M}/z\mathcal{M}$ is called the index of $\mathcal{M}$.

In 1993, Hedenmalm [3] proved the existence of invariant subspaces of $L_a^2(D)$ with index n, $2 \leq n < +\infty$, constructively. In the Hardy space $H^2(D)$, every invariant subspace, except $\{0\}$, has index 1. After Hedenmalm's work, many people have been interested in the structure of invariant subspaces of $L_a^2(D)$, see [4]. In 1996, by Hedenmalm, Richter and Seip [5], invariant subspaces of $L_a^2(D)$ with infinite index were constructed. So, in this paper, we study an invariant subspace of $H^{\infty}(D)$ with infinite index.

Let $H^{\infty} = H^{\infty}(D)$ be the Banach algebra of bounded analytic functions on D. Let $\mathfrak{M} = \mathfrak{M}(H^{\infty})$ be the maximal ideal space of H^{∞} endowed with the weak-$*$ topology. By natural identification, we may consider that $D \subset \mathfrak{M}$. It is known that $\mathfrak{M}$ is a compact Hausdorff space. We identify a function in H^{∞} with its Gelfand transform, so we view H^{∞} as a closed subalgebra of

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