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Extremal odd unimodular lattices in dimensions 44, 46 and 47

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Abstract. In this note, extremal odd unimodular lattices in dimensions 44, 46 and 47 are constructed from self-dual codes over $\mathbb{Z}_4$ and $\mathbb{Z}_6$ by Construction A. The lattices in dimensions 46 and 47 seem to be the first explicit examples for such lattices.

Key words: unimodular lattices, self-dual codes, Construction A.

1. Introduction

A (Euclidean) lattice L is *integral* if $L \subseteq L^*$ where L^* is the dual lattice under the standard inner product $\langle x, y\rangle$. An integral lattice with $L = L^*$ is called *unimodular*. The minimum norm $\min(L)$ of L is the smallest norm among all nonzero vectors of L. Rains and Sloane [8] show that the minimum norm μ of an n-dimensional unimodular lattice is bounded by

$$\mu \leq 2\left[\frac{n}{24}\right] + 2 \tag{1}$$

unless $n = 23$ when $\mu \leq 3$. We say that an n-dimensional (odd) unimodular lattice meeting the bound is called *extremal*. It is a fundamental problem to determine if such a lattice exists for each dimension (cf. [3] and [9]). Conway and Sloane [3] gave the exact bound for the minimum norm of a unimodular lattice of dimensions up to 33. Their work is extended to dimensions 45 except 37, 41, 43 (cf. [9]).

In this note, extremal odd unimodular lattices in dimensions 44, 46 and 47 are constructed by Construction A from self-dual $\mathbb{Z}_6$-codes of length 44, $\mathbb{Z}_4$-codes of lengths 46, 47, respectively. These codes are obtained by considering subtracting from some known extremal self-dual codes of larger lengths. Our lattices in dimensions 46 and 47 seem to be the first explicit examples of extremal ones in these dimensions (cf. [9]).

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