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Averages of Nevanlinna counting functions of holomorphic self-maps of the unit disk

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Abstract. We give an integral representation of the Nevanlinna counting function N_{φ} of a holomorphic self-map φ of the unit disk D in terms of its boundary values φ^*. This representation enables us to explicitly compute the averages of N_{φ} over the circle and over the small disks around the origin. As a consequence, we give, for example, a computational proof of the well known sub-averaging property of N_{φ}.

Key words: Nevanlinna counting function, inner function, sub-averaging property.

1. Introduction

We are only concerned with holomorphic self-maps φ of the open unit disk D on the complex plane. The Nevanlinna counting function

$$N_{\varphi}(w) = \sum_{\varphi(z)=w} \log \frac{1}{|z|}$$

plays a very important role in the holomorphic change of variables by $w = \varphi(z)$ in the integral representation ([ESS], [St]) and in the study of the composition operator $C_{\varphi}(f) = f \circ \varphi$. For example, C_{φ} is a compact operator on the Hardy space H^2 if and only if $N_{\varphi}(w) = o(\log 1/|w|)$. See [Sh1, Sh2]. In this paper, we obtain a representation of N_{φ} in terms of the boundary values φ^* of φ by applying Jensen's formula to $(a-\varphi)/(1-\bar{a}\varphi)$ in Proposition 2.1. It clarifies the behavior of N_{φ} more clearly, and enables us to compute the averages of N_{φ} over the circles and over the small disk around the origin as in Theorem 3.1. The usefulness of such representations is justified by giving a computational proof of the well known sub-averaging property of N_{φ} and by other consequences and the representation of the Nevanlinna counting functions of Rudin's orthogonal functions in Section 4.

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