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First order extensions of holomorphic foliations

(Dedicated to professor Tatsuo Suwa for his 60^{th} birthday)

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Abstract. Let S be a subvariety of a complex manifold M. Let $\mathcal{F}$ be a holomorphic foliation on S and $\mathcal{E}$ a coherent sheaf on S. We give a definition of first order tangency extension of $\mathcal{F}$ to M with respect to $\mathcal{E}$ and prove that, under some suitable hypotheses, the existence of extensions give rise to localization of certain characteristic classes on S. This point of view includes both the classical Camacho-Sad index theorem, variation and the newer indices theorems for holomorphic self-maps along fixed points sets.

Key words: holomorphic foliations, localization of characteristic classes, index theorems.

Introduction

The theory of holomorphic foliations has been studied since the time of Poincaré [16] and Dulac [9]. One of the main question was that of the existence of separatrices through a singular point for a (germ of) one-dimensional foliation in $\mathbb{C}^2$. It has been known since the early years of the past century that "generically" the answer is affirmative. But a final positive answer was obtained only in 1982 by Camacho and Sad [8] who exploited an "index theorem" to reduce the non-generic cases to a known ones. The work of Camacho and Sad gave rise to many studies on those "indices (or residues) theorems". After preliminary works of Lins Neto [15] and Suwa [18], a general comprehension of this phenomenon, together with general principles, is, at least in the opinion of the author, due to Lehmann and Suwa (see, *e.g.*, [12], [13], [14] and [19]) who understood that the Camacho-Sad index theorem and its further generalizations were essentially examples of localizations of characteristic classes of a particular vector bundle due to the existence of a so-called "holomorphic action" on such a bundle outside some closed subsets.

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