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The nullity of a compact minimal hypersurface in a compact symmetric space of rank one

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Abstract. We determine a compact minimal hypersuface with the least nullity in the Cayley projective plane. Combining this with the preceding results, we conclude the following: Let X be a compact symmetric space of rank one and M a compact minimal hypersuface in X. Then the nullity of M is bounded from below by the dimension of X. When the nullity of M is equal to the dimension of X, M must be a minimal geodesic hypersphere in X. Conversely,the nullity of a minimal geodesic hypersphere in X is equal to the dimension of X.

Key words: minimal submanifolds, nullity, Cayley projective space, compact symmetric spaces of rank one..

1. Introduction

In this article, we will prove the following:

Theorem 1.1 *Let M be a compact minimal hypersurface in the Cayley projective plane. Then its nullity satisfies* $\mathrm{nul}(M) \geqq 16$. *When the nullity of M is equal to* 16, *then M must be a minimal geodesic hypersphere.*

Similar results on the nullity of minimal hypersurfaces in spheres were obtained by Simons([S]), in real projective space by Ohnita([O]) and in complex or quaternion projective spaces by the present author([G1], [G2]). Those results are summarized as follows:

Theorem 1.2 *Let X be a compact symmetric space of rank one and M a compact minimal hypersuface in X. Then the nullity of M is bounded from below by the dimension of X. When the nullity of M is equal to the dimension of X, M must be a minimal geodesic hypersphere in X. Conversely, the nullity of a minimal geodesic hypersphere in X is equal to the dimension of X.*

In Section 2, we give a brief review on the Jordan algebra and the group

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