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The Lipschitz continuity of Neumann eigenvalues on convex domains

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Abstract. We consider the Neumann spectrum of the Laplacian on convex domains. Radially parametrizing these domains, we show that each Neumann eigenvalue is Lipschitz continuous with respect to the sup norm on the radial functions. We use this to prove that each Neumann eigenvalue is maximized on the class of convex domains with fixed volume.

Key words: Neumann spectrum, Laplace operator, eigenvalue.

1. Introduction

Suppose $\Omega \subset \mathbb{R}^n$ is a (sufficiently regular) bounded domain, and let $\{\mu_k\}_{k=1}^{\infty}$ be the Neumann spectrum for the Laplacian on Ω (see §2 for the functional analytic characterization of μ_k):

$$\begin{cases} \Delta v + \mu_k v = 0 & \text{on } \Omega, \\ \dfrac{\partial v}{\partial \eta} = 0 & \text{on } \partial\Omega. \end{cases} \tag{1.1}$$

Here, $\mu_1(\Omega) = 0$, corresponding to $v = v_1 \equiv 1$. For $k \geq 2$, we are interested in maximizing $\mu_k \geq 0$ over classes of domains constrained to contain a specific volume V:

$$|\Omega| = V. \tag{V}$$

(The example of long, thin rectangles shows that no minimizer of μ_k exists).

Weinberger proved that amongst domains satisfying (V), μ_2 is maximized by the ball of the appropriate radius ([W]-see also [SY, pp. 140–142], [X, Th2]). For general k, one can easily prove that μ_k is bounded above by k and $|\Omega|$ (see §2). Therefore, it is reasonable to contemplate the existence of - much more ambitiously, the identity of - a maximizer of μ_k amongst

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