

The Weierstrass representation for pluriminimal submanifolds

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Abstract. In this paper a Weierstrass representation formula for pluriminimal submanifolds of the Euclidean space is proposed. Holomorphic 1,0 forms on Kähler varieties are used to globalize the local data. As an application we construct immersions of \mathbb{C}^2 in \mathbb{R}^6 generalizing the example given by Furuhashi. We also show that any affine algebraic variety admits a pluriminimal immersion into some Euclidean space.

Key words: Pluriminimal varieties, Weierstrass representation.

Introduction

The classical methods to describe minimal submanifolds of riemannian manifolds are complex analysis for 2-dimensional domains, and the study of the minimal equation for hypersurfaces. In the intermediate cases these tools do not give a satisfactory description of the picture. It is then natural to restrict the class of minimal submanifolds. For example, when M is a complex manifold of dimension m , (X, g) is a riemannian manifold, following Eschenburg and Tribuzy ([7]) we set:

Definition An immersion $f: M \rightarrow X$ is called *pluriminimal* if the restriction to any smooth complex curve in M is a minimal immersion into X .

We remark that if $m = 1$ pluriminimal is equivalent to minimal.

The first problem is to show that this class of submanifolds contains interesting examples.

In this paper we study the case when (X, g) is the Euclidean space. We propose an analogue of the Weierstrass representation for pluriminimal

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