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Singular perturbation of domains and semilinear elliptic equations III

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Abstract. We consider a (parametrized) bounded domain, some portion of which degenerates and approaches a lower dimensional set when the parameter goes to zero. We consider semilinear elliptic equation with Neumann B.C. in this domain and the behavior of the solutions in the limit. We give a characterization for the solutions in the sense of uniform convergence.

Key words: partial degeneration, semilinear elliptic equation, domain perturbation.

1. Introduction

We deal with a domain $\Omega(\zeta)$ which partially degenerates to a lower dimensional set as $\zeta \to 0$. The shrinking subregion of $\Omega(\zeta)$ is denoted by $Q(\zeta)$. We will consider solutions of the semilinear elliptic equation

$$\Delta u + f(u) = 0 \quad \text{in} \quad \Omega(\zeta), \quad \partial u/\partial\nu = 0 \quad \text{on} \quad \partial\Omega(\zeta) \qquad \text{(Neumann B.C.)} \tag{1.1}$$

for $\zeta \to 0$ and characterize their behaviors in this limitting process. In the previous work [10, 11, 12], we dealt with such problems for the Dumbbell shaped domain (cf. Fig. 1), which is obtained by connecting two disjoint domains by a thin cylindrical channel and gave a characterization of the solutions in the sense of "uniform convergence" when the channel part becomes thinner and approaches a 1-dimensional line segment. The methods of the proofs (scaling technique) used in that work, are restrictive and are not applicable to more general cases of partial degeneration of domains. In this paper we deal with this problem by a direct method based on concrete comparison (barrier) functions and generalize the previous results to the case where the limit set of $Q(\zeta)$ is higher-dimensional (cf. Example, Fig. 2).

There have been several significant studies on reaction-diffusion equations on variable domains since late 70's. Behavior of solutions and their

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