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Remarks on Riesz sets

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§ 1. Introduction.

Let $\hat{G}$ be the dual group of a LCA group G. $M(G)$ denotes the usual Banach algebra of all bounded regular Borel measures on G. $L^1(G)$ is the space of all integrable functions on G with respect to a Haal measure on G. For a $\mu\in M(G)$, its Fourier-Stieltjes transform $\hat{\mu}$ is defined as follows.

$$\hat{\mu}(\gamma)=\int_{G}(-x, \gamma)\,d\mu(x) \qquad \text{for } \gamma\in\hat{G}\,.$$

For a subset E of $\hat{G}$, $M_E(G)$ denotes the subspace of $M(G)$ consisting of measures whose Fourier-Stieltijes transforms vanish off E. Let $G=T^n$, and let P be a positive octant of $\hat{G}=Z^n$. Thar is $P=\{(m_1, \cdots, m_n)\in Z^n\,;\ m_i\geqq 0\ (i=1, \cdots, n)\}$. The following theorem (A) is called the Bochner's theorem.

(A) For every $\mu\in M_P(T^n)$, μ is absolutely continuous with respect to a Lebesque measure on T^n. That is, $M_P(T^n)\subset L^1(T^n)$.

If we exchange T^n by R^n, the same result is established.

The author proved in ([2]) the following theorem.

(B) Let G be a LCA group such that $\hat{G}$ is algebraically ordered. Let $M^a(G)$ denote the subspace of $M(G)$ consisting of measures of analytic type. Suppose $M^a(G)\neq\{0\}$. If $M^a(G)\subset L^1(G)$, then G admits one of the following structures.

(a) $G=R$, (b) $G=R\oplus D$,

(c) $G=T$, (d) $G=T\oplus D$

for some discrete abelian group D. Moreover, let G be one of the above groups. P is a subsemigroup of $\hat{G}$ such that (i) $P\cup(-P)=\hat{G}$ and $P\cap(-P)=\{0\}$. Set $M_P^a(G)=M^a(G)$. Then, $M_P^a(G)\subset L^1(G)$.

We start to consider whether an analogy of the Bochner's theorem is established if we exchange T by $T\oplus D$.

PROPOSITION 1. *Let $H=T\oplus D$, where D is a discrete abelian group such that $\hat{D}$ is torsion-free. Let P_H be a subsemigroup of $\hat{H}=Z\oplus\hat{G}$ such that*