On quadratic first integrals of a particular natural system in classical mechanics

Dedicated to Professor Yoshie Katsurada on her sixtieth birthday

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§ 1. Introduction. Recently, the first author and Kimura [1] have studied a theory of quadratic first integrals in natural systems [2], in connection with the symmetry problem of classical mechanics (e. g., see references in [3], [4] and [5]). As a result, a necessary and sufficient condition has been established for the existence of a quadratic first integral, and the maximum number of linearly independent quadratic first integrals has been found on the basis of this condition.

For the future development of the theory, it seems useful to study quadratic first integrals of some simple dynamical systems. Along this line of thought, Kimura [6] has examined the case of a multidimensional central force, particular attention having been paid to the number of linarly independent quadratic first integrals and to their connection with the linear first integrals.

The present paper is devoted to a similar discussion of another of the simplest systems, that is, the system in which the configuration space is an N-dimensional Euclidean space and the equi-potential surfaces are hyperplanes parallel to each other. This case is of much interest from a mathematical point of view, because the maximum number of linearly independent linear first integrals can be attained only in this case and the central potential case, if the configuration space is taken to be Euclidean [7].

In §§ 2 and 3 the general form of quadratic first integrals is obtained in the system under consideration. In § 4, the number of linearly independent quadratic first integrals is found and the relation between the linear and quadratic first integrals is made clear. Further, the Poisson brackets between the first integrals are calculated with a view to their applications in the symmetry problem. The final section is devoted to a discussion of the results obtained.

§ 2. Basic equations. Let us assume that the configuration space is an N-dimensional Euclidean space referred to Cartesian coordinates x^i and that the potential function U depends on the final coordinate x^N