On periodic orbits of stable flows

Dedicated to Professor Yoshie Katsurada on her 60th birthday

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Introduction. Let M be a compact C^∞ Riemannian manifold of dimension $n\geq 2$ without boundary and X a C^1 vector field on M. Let $\{f^t\}$ be the one-parameter group of C^1-diffeomorphisms f^t of M generated by X. $\{f^t\}$ is called a *differentiable flow* (or dynamical system) on M. More generally, a one-parameter group of homeomorphisms $\{g^t\}$ is called a *continuous flow* on M if the map g: $M\times \mathbf{R}\to M$ defined by $g(x, t)=g^t(x)$ ($x\in M$, $t\in \mathbf{R}$) is continuous.

A point $x\in M$ is called a *periodic point* of $\{f^t\}$ if there is a $t_0>0$ such that $f^{t_0}(x)=x$ holds. We denote by Per $(\{f^t\})$ the set of all periodic points of $\{f^t\}$. The orbit $\{f^t(x)|t\in \mathbf{R}\}$ is called a *periodic orbit* if $x\in$ Per $(\{f^t\})$.

A point $x\in M$ is called a *non-wandering point* of $\{f^t\}$ if for any neighborhood U of x and any $k>0$ we can find a $t_0\geq k$ such that $f^{t_0}(U)\cap U\neq\phi$ holds. We denote by $\Omega(\{f^t\})$ the set of all non-wandering points of $\{f^t\}$. Clearly $\Omega(\{f^t\})$ is closed in M and we have

$$\mathrm{Per}\left(\{f^t\}\right)\subset\Omega\left(\{f^t\}\right).$$

Let Map(M) be the set of all continuous maps f of M into M. For f, $g\in$ Map(M) we define the metric $d(f, g)$ by

$$d(f, g)=\sup_{x\in M} d\left(f(x), g(x)\right),$$

where d denotes the metric on M induced by the Riemannian metric on M. For any continuous function μ on M we define the norm $\|\mu\|$ by

$$\|\mu\|=\operatorname*{Max}_{x\in M}\left|\mu(x)\right|.$$

DEFINITION 1. $\{f^t\}$ is called to be *topologically stable*, if there exists a positive number ε_0 having the following property: For any positive $\varepsilon<\varepsilon_0$, there exists a positive $\delta=\delta(\varepsilon)$ such that for any continuous flow $\{g^t\}$ with $d(f^t, g^t)<\delta$ for $t\in\left[\frac{1}{4}, 1\right]$, there exist a continuous function p on $M\times\mathbf{R}$ and a surjective map $u\in$ Map(M) such that

$$u\left(g^t(x)\right)=f^{p(x,t)}\left(u(x)\right)$$