Finsler spaces as distributions on Riemannian manifolds

Dedicated to Professor Yoshie Katusrada on her Sixtieth Birthday

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§ 1. Introduction. In the previous paper [6][1], on making use of the methods in the classical theories and the method due to M. Kurita [3], [4] we have studied a Finsler space V_n with the following fundamental function: $F=\sqrt{g_{ij}y^i y^j}+\alpha_i y^i$. Especially we have shown that the connection of E. Cartan can give rise to the affine connections on the p-manifold N of V_n in the theory of M. Kurita [4] and that the space V_n and its geometry are realizable in the N.

The principal purpose of the present paper is to show that the above two facts hold good also in a general Finsler space with the fundamental metric function of class C^4. As a consequence we have that this leads to the theory of A. Deicke [1], [2] and suggests a new method to study Finsler spaces.

§ 2. Contact structure. Let M be an n-dimensional paracompact differentiable manifold and x^i be local coordinates in a neighborhood U of any point $x\in M$. In the tangent space T_x and the dual tangent one T_x^* at x, we take a natural frame (e_i) and its dual one (e^i), and denote by y^i and p_i the components of any vectors y and p in T_x, T_x^* respectively. Further we consider the tangent bundle TM and the dual tangent one T^*M over M. We assume that M is endowed with a metric function $F(x, y)$ satisfying the following conditions;

(2.1)
(1) $F(x, y)$ is of class C^4 and is positively homogeneous of degree 1 in the y^i.
(2) $F(x, y)$ is positive if not all y^i vanish simultaneously.
(3) $g_{ij}(x, y)Z^i Z^j$ is positive definite,

where $g_{ij}(x, y)=\dfrac{1}{2}\dfrac{\partial^2 F^2}{\partial x^i \partial x^j}$.

Now we consider a mapping $\varphi: TM\to T^*M$ defined by $(x, y)\to(x, p)$ with

$$p_i=\frac{\partial F}{\partial y^i} \qquad (i=1, 2, \cdots, n). \tag{2.2}$$

1) Numbers in brackets refer to the references at the end of the paper.