On certain integral formulas for hypersurfaces in a constant curvature space

Dedicated to Professor Yoshie Katsurada on her 60th birthday

By Tamao NAGAI

§ 0. Introduction.

Let V^m be a closed orientable hypersurface twice differentiably imbedded in an $(m+1)$-dimensional Euclidean space $E^{m+1}(m+1\geqq 3)$ and $k_1, \cdots, k_m$ be the m principal curvatures at a point P of V^m. The ν-th mean curvature H_ν of V^m at P is defined by

$$\binom{m}{\nu} H_\nu = \sum k_1 \cdots k_\nu \qquad (\nu=1, 2, \cdots, m),$$

where the right hand member denotes the ν-th elementary symmetric function of $k_1, \cdots, k_m$. It is convenient to define $H_0=1$. C. C. Hsiung [1][1)] proved

$$\int_{V^m} (H_{\nu+1} p + H_\nu) dA = 0 \qquad (\nu=0, 1, \cdots, m-1), \tag{0.1}$$

where p denotes the oriented distance from a fixed point O in E^{m+1} to the tangent space of V^m at P and dA is the area element of V^m. Let $\bar{V}^m$ be a closed orientable hypersurface parallel to the given V^m. Then, the integral formulas (0.1) have been derived by comparison between associated quantities of V^m and $\bar{V}^m$.

Let R^{m+1} be an $(m+1)$-dimensional Riemann space of class $C^r (r\geqq 3)$, which admits an infinitesimal conformal transformation

$$\bar{x}^i = x^i + \xi^i(x)\delta\tau . \tag{0.2}$$

We assume that a closed orientable hypersurface V^m does not pass through any singular point of a tangent vector field of the paths with respect to the infinitesimal transformation (0.2). Since the transformation is conformal, there exists a scalar field Φ and the vector ξ^i satisfies the relation

$$\xi_{i;j} + \xi_{j;i} = 2\Phi g_{ij} , \tag{0.3}$$

where $\xi_i = g_{ij}\xi^j$ and the symbol ";" means covariant differentiation with respect to Riemann connection determined by the metric tensor g_{ij} of R^{m+1}

1) Numbers in brackets refer to the references at the end of the paper.