# PARTIALLY ORDERED ABELIAN SEMIGROUPS II. ON THE STRONGNESS OF THE LINEAR ORDER DEFINED ON ABELIAN SEMIGROUPS

By

Osamu NAKADA

In my previous paper[1], I note that a strong linearly ordered abelian semigroup (strong l.o. semigroup) is always normal, its elements are of infinite order except the unit element (if there exists) and the product cancellation law is held in it. In this Part, I shall seek for the other conditions which are exchangeable for the cancellation law. But the some theorems in this note will be expressed in the form of a partially ordered abelian semigroup.

*Definition 1.* Let $a$ be any element of an abelian semigroup $S$. We denote the set of all positive powers of $a$, in other words, the sub-semigroup of $S$ generated by $a$, by $S(a)$, which is called the *sector* by $a$. And let $T(a)$ be the set of all elements of $S$ whose some positive powers belong to $S(a)$, that is, the set of all elements $x$ of $S$ such that $x^m = a^n$ for some positive integers $m$ and $n$. $T(a)$ is called the *complete sector* by $a$.

Let $a$ and $b$ be any two elements of $S$. Then the two complete sectors $T(a)$ and $T(b)$ are either disjoint or identical. Hence, $S$ is the union of mutually disjoint complete sectors, and clearly $T(a)$ is a sub-semigroup of $S$.

*Definition 2.*[2] Let $a$ be any element of an abelian semigroup $S$, and we shall consider the sector $S(a)$ by $a$. There are two possible cases.

(1) Partially ordered abelian semigroups. I. On the extension of the strong partial order defined on abelian semigroups. Journal of the Faculty of Science, Hokkaido University, Series I, vol. XI, No. 4 (1951) pp. 181–189; this is referred to hereafter as "O.I."

A set $S$ is said to be a *partially ordered abelian semigroup* (p.o. semigroup), when $S$ is (I) an abelian semigroup (not necessarily contains the unit element), (II) a partially ordered set, and satisfies (III) the *homogeneity*: $a \geqq b$ implies $ac \geqq bc$ for any $c$ of $S$. When a partial order $P$ of an abelian semigroup $S$ satisfies the condition (III), we call $P$ a *partial order defined on* an abelian semigroup $S$. (Definition 1, O.I.)

(2) Cf. D. REES: On semi-groups, Proc. Cambridge Phil. Soc., vol. 36 (1940), pp. 387–400.