REMARK ON THE STRUCTURE OF LIE AND JORDAN RINGS

By

Masahiko ATSUCHI

The purpose of this paper[1] is to investigate the form of multiplication of Lie and Jordan rings when we seek for their model in a non-commutative ring.

Suppose L is the module with operator in which a multiplication is defined and the following conditions are satisfied:

(1) $a \times (b+c) = a \times b + a \times c, \quad (b+c) \times a = b \times a + c \times a$

(2) $\lambda (a \times b) = (\lambda a) \times b = a \times (\lambda b), \quad a, b, c \in L, \quad \lambda \in P,$

where P is a operator domain.

Suppose that P is the field whose characteristic is $m \, (0 < m \leqq \infty)$ (in our case we say the characteristic of a field is ∞ when it is 0 in usual sense), and S is the non-commutative ring in which L is mapped as the module by an operator-homomorphism φ.

Now we assume that $a \times b$ is mapped on the polynomial with coefficients in P of degree $n \, (< m)$ with respect to x, y which are the images of a, b by the homomorphism φ respectively:

$$\varphi (a \times b) = \sum \xi^{i_1, \cdots, i_r}_{j_1, \cdots, j_r} x^{i_1} y^{j_1} \cdots x^{i_r} y^{j_r} + C,$$

$$\xi \in P, \quad x, y, C \in S, \quad i_1 + \cdots + i_r \leqq n, \quad i_k \geqq 0,$$

$$j_1 + \cdots + j_r \leqq n, \quad j_k \geqq 0, \quad i_1 + \cdots + i_r + j_1 + \cdots + j_r \neq 0,$$

where $\sum$ covers $i_1, \cdots, i_r, j_1, \cdots, j_r$ satisfying the above conditions.

Being $0 \times b = a \times 0 = 0 \times 0 = 0$, it follows immediately that C and the sum of all terms with $i_1 + \cdots + i_r = 0$ or $j_1 + \cdots + j_r = 0$ vanish. Therefore we can assume from the beginning that

$$\varphi (a \times b) = \sum \xi^{i_1, \cdots, i_r}_{j_1, \cdots, j_r} x^{i_1} y^{j_1} \cdots x^{i_r} y^{j_r}, \qquad \begin{matrix} i_1 + \cdots + i_r > 0 \\ j_1 + \cdots + j_r > 0 \end{matrix}.$$

---

(1) This paper has been completed by the encouragement of Prof. M. MORIYA and the helps of my colleagues.