PARTIALLY ORDERED ABELIAN SEMIGROUPS

I. ON THE EXTENSION OF THE STRONG PARTIAL ORDER DEFINED ON ABELIAN SEMIGROUPS

By

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Definition 1. A set S is said to be a *partially ordered abelian semigroup* (p. o. semigroup), when in S are satisfied the following conditions:

I) S is an abelian semigroup under the multiplication, that is:
1) A single-valued product ab is defined in S for any pair a, b of S,
2) $ab = ba$ for any a,b of S,
3) $(ab)\,c = a\,(bc)$ for any a, b, c of S.

II) S is a partially ordered set under the relation $\geqq$, that is:
1) $a \geqq a$,
2) $a \geqq b$, $b \geqq a$ imply $a = b$,
3) $a \geqq b$, $b \geqq c$ imply $a \geqq c$.

III) Homogeneity: $a \geqq b$ implies $ac \geqq bc$ for any c of S.

A partial order which satisfies the condition III) is called a *partial order defined on an abelian semigroup.*

If S is an abelian group, then S is said to be a *partially ordered abelian group* (p. o. group).

Moreover, if a partial order defined on an abelian semigroup (group) S is a linear order, then S is said to be a *linearly ordered abelian semigroup* (*group*) (l. o. semigroup (l. o. group)).

We write $a > b$ for $a \geqq b$ and $a \neq b$.

Definition 2. A partial order defined on an abelian semigroup S (or a p. o. semigroup S) is called *strong,* when the following condition is satisfied: $ac \geqq bc$ implies $a \geqq b$.

Theorem 1. A partial order defined on an abelian group G is always strong.

Proof. Since G is a group, there exists an inverse element c^{-1} of c. By the homogeneity $ac \geqq bc$ implies $(ac)\,c^{-1} \geqq (bc)\,c^{-1}$. Therefore $a \geqq b$.