# A CHARACTERIZATION OF THE MODULARS OF $L_p$ TYPE

By

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A modular on a universally continuous semi-ordered linear space $R$ is, as defined in [1], a functional $m(x)\,(x \in R)$ satisfying the following conditions:

(i) $0 \leqq m(x) \leqq +\infty$, $m(0)=0$;

(ii) $m(\xi x)$ is a convex function of $\xi$ which is finite in a neighbourhood of 0 and not identically zero, if $x \neq 0$;

(iii) $|x| \leqq |y|$ implies $m(x) \leqq m(y)$;

(iv) $x \perp y$ implies $m(x+y) = m(x)+m(y)$;

(v) $0 \leqq x_\lambda \uparrow_{\lambda \in \Lambda} x$ implies $m(x) = \sup_{\lambda \in \Lambda} m(x_\lambda)$.

Since the set of elements $\{x\colon m(x) \leqq 1\}$ is convex, we can define a norm $|||x|||$ such that $|||x||| \leqq 1$ is equivalent to $m(x) \leqq 1$. This norm is said to be the *modular norm.* On the other hands, putting

$$\|x\| = \inf_{\xi > 0} \frac{1+m(\xi x)}{\xi},$$

we obtain another norm which is conjugate to the modular norm of the conjugate modular in case that the space $R$ is semi-regular. We have a relation between these two norms, that is,

$$|||x||| \leqq \|x\| \leqq 2|||x|||.$$

In the space $L_p\,(p \geqq 1)$, putting

$$m(x) = \int_0^1 |x(t)|^p \, dt,$$

we obtain a modular and we have in this case

(1) $$m(x) = |||x|||^p$$

and

(2) $$\|x\| = \alpha |||x|||,$$

where $\alpha$ is the number such that