FOURIER SERIES IX: STRONG SUMMABILITY OF THE DERIVED FOURIER SERIES.

By

Shin-ichi IZUMI and Masakiti KINUKAWA

1. Introduction. Let $f(x)$ be an integrable and periodic function with period 2π and its Fourier series and its conjugate be

$$(1.1)\qquad a_0/2+\sum_{n=1}^{\infty}(a_n\cos nx+b_n\sin nx)\equiv\sum_{n=0}^{\infty}A_n(x)\ ,$$

$$(1.2)\qquad \sum_{n=1}^{\infty}(b_n\cos nx-a_n\sin nx)\equiv\sum_{n=1}^{\infty}B_n(x)\ .$$

Further, let their termwise derived series be

$$(1.3)\qquad \sum_{n=1}^{\infty}n\,(b_n\cos nx-a_n\sin nx)=\sum_{n=1}^{\infty}nB_n(x)\ ,$$

$$(1.4)\qquad -\sum_{n=1}^{\infty}n\,(a_n\cos nx+b_n\sin nx)=-\sum_{n=1}^{\infty}nA_n(x)\,.$$

A series $\sum_{n=1}^{\infty}c_n$ is said to be summable H_k or strongly summable to s, if

$$\sum_{n=0}^{m}|s_n-s|^k=o(m)\qquad (m\to\infty)\ ,$$

where $s_n=\sum_{k=0}^{n}c_k$.

B. N. Prasad and U. N. Singh [7] have found a criteria for H_1 summability of the derived Fourier series which reads as follows:

Theorem A. *If $f(t)$ is a continuous function of bounded variation and if for some $\alpha>1$,*

$$(1.5)\qquad G(t)=\int_0^t|dg(u)|=o\left\{t\Big/\left(\log\frac{1}{t}\right)^{\alpha}\right\},\qquad as\quad t\to 0\ ,$$

where $\qquad g(u)=g_x(u)=f(x+u)-f(x-u)-2us\ ,$
then

$$\sum_{n=1}^{m}|\tau_n(x)-s|=o(m)\ ,$$