A GENERALIZATION OF MAZUR-ORLICZ THEOREM ON FUNCTION SPACES

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1. Introduction. Let $\Omega(\boldsymbol{B}, \mu)$ be a locally finite[1] measure space. By many investigators various function spaces consisting of locally almost finite $\boldsymbol{B}$-measurable functions[2] on Ω have been considered as a generalization of the so-called $\boldsymbol{L}_p$-spaces on Ω $(1 \leqq p \leqq +\infty)$. One of them is $\boldsymbol{L}_{M(u,\omega)}$-space (Musielak-Orlicz [3], [4]).

Let $M(u, \omega)$ be a function on $[0, +\infty] \times \Omega$ with the following properties (it will be called (M)-*function*);

(M)
1) $0 \leqq M(u, \omega) \leqq +\infty$ for all $(u, \omega) \in [0, +\infty] \times \Omega$,
2) $\lim_{u \to 0} M(u, \omega) = 0$ for all $\omega \in \Omega$,
3) $M(u, \omega)$ is a non-decreasing and left continuous[3] function of u for all $\omega \in \Omega$,
4) $\lim_{u \to \infty} M(u, \omega) > 0$ for all $\omega \in \Omega$,
5) $M(u, \omega)$ is locally $\boldsymbol{B}$-measurable[4] as a function of ω for all $u \in [0, +\infty]$.

Using this function $M(u, \omega)$ we can define a functional $\rho_M(x)$ on locally almost finite $\boldsymbol{B}$-measurable functions $x(\omega)$ $(\omega \in \Omega)$ by the formula

$$\rho_M(x) = \int_\Omega M[|x(\omega)|, \omega] d\mu \tag{1}$$ [5]

If $\boldsymbol{L}_{M(u,\omega)}$ denotes the set of all $x(\omega)$ such that $\rho_M(\alpha x) < +\infty$ for a positive number $\alpha = \alpha(x)$ depending on x, $\boldsymbol{L}_{M(u,\omega)}$ is a vector space.

As special cases, $\boldsymbol{L}_{M(u,\omega)}$ coincides with four typical spaces respectively:

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1) Ω is covered by the family of measurable sets of finite measure.

2) Correctly speaking, we shall consider only the functions which are almost finite real valued and $\boldsymbol{B}$-measurable in every measurable set of finite measure. And two functions $x(\omega)$ and $y(\omega)$ are identified if $x(\omega) = y(\omega)$ except on a set of measure zero in every measurable set of finite measure.

3) Since $M(u, \omega)$ can be replaced by $M(u-0, \omega)$, the left side continuity is not essential for the definition of the space $\boldsymbol{L}_{M(u,\omega)}$.

4) It is unnecessary for $M(u, \omega)$ to be almost finite valued.

5) (M)-2) and 3) imply the measurability of a function $M[|x(\omega)|, \omega]$. The integration on Ω means the supremum of integrations on every finite measured set.