ON F-NORMS OF QUASI-MODULAR SPACES

By

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§1. Introduction. Let R be a *universally continuous semi-ordered linear space* (*i.e. a conditionally complete vector lattice* in Birkhoff's sense [1]) and ρ be a functional which satisfies the following four conditions:

(ρ.1) $0 \leqq \rho(x) = \rho(-x) \leqq +\infty$ for all $x \in R$;

(ρ.2) $\rho(x+y) = \rho(x) + \rho(y)$ for any $x, y \in R$ with $x \perp y$[1];

(ρ.3) If $\sum_{\lambda \in \Lambda} \rho(x_\lambda) < +\infty$ for a mutually orthogonal system $\{x_\lambda\}_{\lambda \in \Lambda}$[2], there exists $x_0 \in R$ such that $x_0 = \sum_{\lambda \in \Lambda} x$ and $\rho(x_0) = \sum_{\lambda \in \Lambda} \rho(x_\lambda)$;

(ρ.4) $\overline{\lim_{\xi \to 0}} \rho(\xi x) < +\infty$ for all $x \in R$.

Then, ρ is called a *quasi-modular* and R is called a *quasi-modular space*.

In the previous paper [2], we have defined a quasi-modular space and proved that if R is a non-atomic quasi-modular space which is semi-regular, then we can define a modular[3] m on R for which every universally continuous linear functional[4] is continuous with respect to the norm defined by the modular[5] m [2; Theorem 3.1].

Recently in [6] J. Musielak and W. Orlicz considered a modular $\boldsymbol{\rho}$ on a linear space L which satisfies the following conditions:

(A.1) $\boldsymbol{\rho}(x) \geqq 0$ and $\boldsymbol{\rho}(x) = 0$ if and only if $x = 0$;

(A.2) $\boldsymbol{\rho}(-x) = \boldsymbol{\rho}(x)$;

(A.3) $\boldsymbol{\rho}(\alpha x + \beta y) \leqq \boldsymbol{\rho}(x) + \boldsymbol{\rho}(y)$ for every $\alpha, \beta \geqq 0$ with $\alpha + \beta = 1$;

(A.4) $\alpha_n \to 0$ implies $\boldsymbol{\rho}(\alpha_n x) \to 0$ for every $x \in R$;

(A.5) for any $x \in L$ there exists $\alpha > 0$ such that $\boldsymbol{\rho}(\alpha x) < +\infty$.

They showed that L is a quasi-normed space with a quasi-norm $\|\cdot\|_0$ defined by the formula;

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1) $x \perp y$ means $|x| \frown |y| = 0$.

2) A system of elements $\{x_\lambda\}_{\lambda \in \Lambda}$ is called *mutually orthogonal*, if $x_\lambda \perp x_\gamma$ for $\lambda \neq \gamma$.

3) For the definition of a modular, see [3].

4) A linear functional f is called *universally continuous*, if $\inf_{\lambda \in \Lambda} f(a_\lambda) = 0$ for any $a_\lambda \downarrow_{\lambda \in \Lambda} 0$. R is called *semi-regular*, if for any $x \neq 0$, $x \in R$, there exists a universally continuous linear functional f such that $f(x) \neq 0$.

5) This modular ρ is a generalization of a modular m in the sense of Nakano [3 and 4]. In the latter, there is assumed that $m(\xi x)$ is a convex function of $\xi \geqq 0$ for each $x \in R$.