ON A SIMPLE RING WITH A GALOIS GROUP OF ORDER p^e

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Recently in [2, §3],[1] the next was obtained: *Let R be a simple ring (with minimum condition) of characteristic $p\neq 0$, and $\mathfrak{G}$ a DF-group of order p^e. If $S=J(\mathfrak{G}, R)$, then $[R:S]$ divides p^e, and $V_R(S)$ coincides with the composite of the center of R and that of S.* More recently, in [1], M. Moriya has proved the following: *Let R be a division ring, $\mathfrak{G}$ an automorphism group*[2] *of order p^e (p a prime), and $S=J(\mathfrak{G}, R)$. If the center of S contains no primitive p-th roots of* 1, *then $[R:S]$ divides p^e, and $V_R(S)$ coincides with the composite of the center of R and that of S. And moreover, $[R:S]$ is equal to p^e provided R is not of characteristic p.* The purpose of this note is to extend these facts to simple rings in such a way that our extension contains also the fact cited at the beginning.

In what follows, we shall use the following conventions: R is a simple ring with the center C, and $\mathfrak{G}$ a DF-group of order p^e where p is a prime number. We set $S=J(\mathfrak{G}, R)$, which is a simple ring by [2, Lemma 2]. And by Z and V we shall denote the center of S and the centralizer $V_R(S)$ of S in R respectively. Finally, as to notations and terminologies used here, we follow [2].

Now, we shall begin our study with the following theorem.

Theorem 1. *If Z contains no primitive p-th roots of* 1, *then $[R:S]$ diviees p^e.*

Proof. Firstly, in case $e=1$, $\mathfrak{G}$ is either outer or inner. If $\mathfrak{G}$ is outer, then it is well-known that there holds $[R:S]=p$. Thus, we may, and shall, assume that $\mathfrak{G}$ is inner, and set $\mathfrak{G}=\{1, \tilde{v}, \cdots, \tilde{v}^{p-1}\}$. Then, to be easily seen, v is contained in $Z(\supseteq C)$, and $v^p=c$ for some $c\in C$. If the polynomial $X^p-c\in C[X]$ is reducible, then it possesses a linear factor, that is, there exists an element $c_0\in C$ such that $c_0^p=c$, whence it follows that

1) Numbers in brackets refer to the references cited at the end of this note.

2) One may remark here that in case R is a division ring any automorphism group of finite order becomes naturally a DF-group.