ON AUTOMORPHISM GROUPS OF FINITE ORDER IN DIVISION RINGS

By

Hisao TOMINAGA

It is a well-known theorem of E. Artin that if F is an algebraically closed (commutative) field of characteristic 0 then any automorphism of F of finite order is of order at most 2. Recently, in [1], H. Lenz drew out the essence of Artin's proof [1, Satz 3], and obtained several results concerned with automorphism groups of finite order. In the present note, we shall prove an extension of [1, Satz 3] to division rings and that of [1, Satz 5], whose proof is notably easy.

Let K be a division ring with the center Z, and $\mathfrak{G}$ an automorphism group of K. Then, $\mathfrak{G}$ induces an automorphism group $\overline{\mathfrak{G}}$ of the group K^*/Z^*, where K^* is the multiplicative group consisting of all non-zero elements of K. Particularly, if $\overline{\mathfrak{G}}$ coincides with the identity group then $\mathfrak{G}$ will be called an *M-group*. In case K is commutative, the notion of M-group is trivial of course. Now let p be a prime number, and k an element of K. If there exists a division subring K' of K such that $k^p \in K'$ and $k \notin K'$, then k will be called a *p-th root of* K. We consider here the following property of K:

(P) *For each p-th root k of K, the equation* $x^p - k = 0$ *has a solution in K.*

At first, we shall prove the following fundamental lemma.

Lemma 1. *Let K be strictly Galois*[1] *with respect to an M-group* $\mathfrak{P} = \{\sigma^i\}$ *of order p, and possess the property* (P). *If Z contains a primitive* p^2*-th root* η *of* 1, *then* η *is not contained in* $L = J(\mathfrak{P}, K)$ (*=the fix-subring of* $\mathfrak{P}$).

Proof. Evidently, $\zeta = \eta^p$ is a primitive p-th root of 1, and $[\Phi(\zeta):\Phi] < p$, where Φ is the prime subfield of K. If $\zeta\sigma \neq \zeta$, then $\Phi(\zeta)$ being $\mathfrak{P}$-normal, we have $[\Phi(\zeta):\Phi] \geq p$. We see therefore ζ is contained in L. Accordingly, by [2, Corollary 2], there exists a non-zero element $x \in K$ such that $x\sigma = x\zeta$. As $x^p \in L$ and $K = L[x]$ consequently, the property (P)

1) Cf. [3].