# PARTIALLY ORDERED ABELIAN SEMIGROUPS. III

## ON THE REVERSIBLE PARTIAL ORDER DEFINED ON AN ABELIAN SEMIGROUP

By

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In their paper,[1] Ben Dushnik and E. W. Miller introduced the concept of the reversible partial order and expressed the theorem about this concept. In this Part III, I shall show that the same one is held in the partially ordered abelian semigroup by adding the certain condition.

**Definition 1.** A set $S$ is said to be a *partially ordered abelian semigroup* (p.o. semigroup), when $S$ is (I) an abelian semigroup (not necessarily contains the unit element), (II) a partially ordered set, and satisfies (III) the *homogeneity*: $a \geqq b$ implies $ac \geqq bc$ for any $c$ of $S$.

A partial order which satisfies the condition (III) is called a *partial order defined on an abelian semigroup.*

Moreover, if a partial order defined on an abelian semigroup $S$ is a linear order, then $S$ is said to be a *linearly ordered abelian semigroup* (l.o. semigroup). (Definition 1, O.I.)

**Definition 2.** Let $\mathfrak{S}=\{P_\alpha\}$ be any set of partial orders, each defined on the same abelian semigroup $S$. We define the new partial order $P$ on $S$ as follows: For any two elements $a, b$, we put $a \geqq b$ in $P$ if and only if $a \geqq b$ in every $P_\alpha$ of the set $\mathfrak{S}$. Indeed, $P$ is again a partial order defined on $S$. This partial order $P$ is said to be the *product* of the partial orders $P_\alpha$ or to be *realized* by the set $\mathfrak{S}$ of partial orders $P_\alpha$. (Definition 9, O.I.)

By the *dimension* of a partial order $P$ defided on an abelian semigroup $S$ is meant the smallest cardinal number $\mathfrak{m}$ such that $P$ is realized by $\mathfrak{m}$ linear orders defined on $S$.

**Definition 3.** Let $P$ and $Q$ be two partial orders defined on the same

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Partially ordered abelian semigroup. I. On the extension of the strong partial order defined on abelian semigroups. Journ. Fac. Sci., Hokkaido University, Series I, vol. XI (1951), pp. 181–189; this is referred to hereafter as "O.I."

1) Ben Dushnik and E. W. Miller: Partially ordered sets, Amer. Math. Journ. vol. 63 (1941), pp. 600–610.