ON $*$-MODULAR RIGHT IDEALS OF AN ALTERNATIVE RING

By

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It is well known that, in any alternative ring A, the Smiley radical $SR(A)$ is contained in every modular maximal right ideal M. E. Kleinfeld has shown that every primitive alternative, non-associative ring is a Cayley-Dickson algebra.

Now we introduce the notion of $*$-modularity as follows: a right ideal I of an alternative ring A is called $*$-modular if there exist two elements $a, u \in A$ such that

$$(1) \qquad x+ax+(a, x, u) \in I$$

for all $x \in A$, where (a, x, u) denotes the associator $ax \cdot u - a \cdot xu$ of a, x, u, and in this case we call a a *left $*$-modulo unit of I.* Clearly, modularity implies $*$-modularity.

In this note, we shall show that the above results are also true if we replace modular ideals by $*$-modular ideals.

If a ring A is assumed to be alternative, then (a, b, c) becomes a skew-symmetric function of its three variables.

The Smiley radical $SR(A)$ of an alternative ring A is defined as the totality of elements $z \in A$ for which each element of $(z)_r$ is right quasi-regular.

In the next lemma we develop an important property of $*$-modular right ideals.

Lemma 1. *Let I^* be a $*$-modular right ideal of an alternative ring A, and suppose that a left $*$-modulo unit a of I^* is right quasi-regular. Then $I^*=A$.*

Proof. Let b be a right quasi-inverse of a:

$$(2) \qquad a+b+ab=0 .$$

Since a is a left $*$-modulo unit of I^* and since $(a, a, u)=0$, we have $a+a^2 \in I^*$ by putting $x=a$ in (1), while $(a+a^2)b-(a, b, u)=ab+a^2b-(a, a, u)-(a, b, u)=ab+a^2b-(a, a+b, u)=ab+a^2b+(a, ab, u) \in I^*$ by (2). Hence it follows that $(a, b, u) \in I^*$. On the other hand, if we put $x=b$ in (1), we