ON GROUPS OF ROTATIONS IN MINKOWSKI SPACE II

By

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§ 1. **Introduction.** The present paper is a continuation of the one with the same title "On groups of rotations in Minkowski space I." [7][1] Let M_{n+1} be an $n+1$-dimensional Minkowski space whose indicatrix I_n is defined by $F(X^1, X^2, \cdots, X^{n+1})=1$. As I_n is an n-dimensional hypersurface in M_{n+1}, we may represent I_n by $n+1$ equations involving n parameters as

$$X^i=X^i(u^\alpha). \quad (i=1, 2, \cdots, n+1;\ \alpha=1, 2, \cdots, n)^{2)}$$

Putting $A_{ijk}(X)=F(X)C_{ijk}$, where $C_{ijk}=\frac{1}{2}\frac{\partial g_{ij}}{\partial X^k}$ and $g_{ij}=\frac{1}{2}\partial^2 F/\partial X^i \partial X^j$, A_{ijk} is a symmetric covariant tensor in M_{n+1}. If we denote by $g_{\alpha\beta}$ and $A_{\alpha\beta\gamma}$ the induced components of g_{ij} and A_{ijk} respectively, i.e.

$$g_{\alpha\beta}=g_{ij}X^i_\alpha X^j_\beta, \quad A_{\alpha\beta\gamma}=A_{ijk}X^i_\alpha X^j_\beta X^k_\gamma \quad (X^i_\alpha=\partial X^i/\partial u^\alpha),$$

I_n may be considered as an n-dimensional compact Riemannian space with the fundamental metric tensor $g_{\alpha\beta}$, and the Riemannian space is characterized by existence of a symmetric covariant tensor $A_{\alpha\beta\gamma}$. Moreover, it is remarkable that in I_n we have

$$R_{\alpha\beta\gamma\delta}=S_{\alpha\beta\gamma\delta}+(g_{\alpha\gamma}g_{\beta\delta}-g_{\alpha\delta}g_{\beta\gamma}), \tag{1.1}$$

where $R_{\alpha\beta\gamma\delta}=g_{\alpha\varepsilon}R^{\varepsilon}_{\cdot\beta\gamma\delta}$ and

$$R^{\alpha}_{\cdot\beta\gamma\delta}=\frac{\partial\{{}^{\alpha}_{\beta\delta}\}}{\partial u^\gamma}-\frac{\partial\{{}^{\alpha}_{\beta\gamma}\}}{\partial u^\delta}+\left\{{\varepsilon \atop \beta\delta}\right\}\left\{{\alpha \atop \varepsilon\gamma}\right\}-\left\{{\varepsilon \atop \beta\gamma}\right\}\left\{{\alpha \atop \varepsilon\delta}\right\},$$

$$S_{\alpha\beta\gamma\delta}=A_{\alpha\delta}^{\cdot\cdot\sigma}A_{\sigma\gamma\beta}-A_{\alpha\gamma}^{\cdot\cdot\sigma}A_{\sigma\delta\beta}.$$

Also, $A_{\alpha\beta\gamma}$ satisfies the relation $A_{\alpha\beta\gamma;\delta}=A_{\alpha\beta\delta;\gamma}$ [6].[3]

When a centro-affine transformation in M_{n+1}, with its centre at origin, preserves I_n, the transformation is called a *rotation* in M_{n+1}.

1) Numbers in brackets refer to the references at the end of the paper.

2) Throughout the present paper, the Greek indices $\alpha, \beta, \gamma, \cdots$ are supposed to run over the range $1, 2, \cdots, n$.

3) Semi-colon is used to represent the covariant differentiation with respect to the Christoffel symbols made by $g_{\alpha\beta}$.