ON GROUPS OF ROTATIONS IN MINKOWSKI SPACE I

By

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§ 1. **Introduction.** Let F_{n+1} be an $n+1$-dimensional Finsler space with the fundamental function $F(x^i, x'^i)$ $(i=1, 2, \cdots, n+1)$.[1] At every point with coordinate (x_0^i) in F_{n+1}, we obtain an $n+1$-dimensional Minkowski space M_{n+1}, whose indicatrix I_n is given by the end points of the vectors (X^i)'s at the origin (x_0^i) satisfying the equation

$$g_{ij}(x_0, X)X^iX^j=1 \quad \left(g_{ij}(x, X)=\frac{1}{2}\partial^2F(x, X)\Big/\partial X^i\partial X^j\right). \tag{1.1}$$

At any point (x_0^i), the quadratic form for any fixed vector X_0^i:

$$g_{ij}(x_0, X_0)X^iX^j=1 \tag{1.2}$$

defines a hyperquadric I_n^* which is in double contact with I_n at two points of coordinates (X_0^i) and $(-X_0^i)$ respectively.

L. Berwald [5],[2] E. Cartan [6] and many others regarded a Finsler space as a space of line-elements (x^i, x'^i). From this point of view, we can obtain for each line-element $(x_0^i, x_0'^i)$ an $n+1$-dimensional tangent Euclidean space $E_{n+1}(x_0^i, x_0'^i)$ whose indicatrix is a hyperquadric I_n^* determined by (1.2) by putting $X_0^i=x_0'^i$. Under this consideration the connection in F_{n+1} was established by defining a suitable correspondence between neighbouring tangent Euclidean spaces $E_{n+1}(x^i, x'^i)$ and $E_{n+1}(x^i+dx^i, x'^i+dx'^i)$.

Recently W. Barthel [1]–[4], A. Kawaguchi [9], D. Laugwitz [10] and H. Rund [11] reconstructed the foundation of the theory of Finsler space from the stand point that the Finsler space is a point space but is not a line-element space, that is to say, the tangent space at each point (x_0^i) in F_{n+1} should be regarded as a Minkowski space with an Indicatrix determined by $F(x_0^i, X^i)=1$. On account of this fact, in order to establish the theory of Finsler space, it becomes an important problem to study

1) For the sake of convenience, we suppose that the dimension of a Finsler space is $n+1$, because in the present paper we shall discuss mainly about the theory of transformations in an n-dimensional indicatrix I_n.

2) Numbers in brackets refer to the references at the end of the paper.