ON THE HILBERT TRANSFORM I*)

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1. Introduction. The theory of Hilbert transform has been studied by many authors. Its references will be found in the book of E. C. Titchmarsh [30]. Recently A. P. Calderón-A. Zygmund [5] show that this classical theory can be treated by another method in the n-dimensional Euclidean space. The foundation of their arguments is the interpolation of linear operation. There are several studies of J. Marcinkiewicz in Fourier series from this point of views. In particular he has presented a brief note in Comptes rendus vol. 208 (1939), 1271–1273, without proof. Recently A. Zygmund [36], a teacher of his, has completed these theorems.

In chapter 1, we shall extend one of these theorems on the totally σ-finite measure space in a sense of P. R. Halmos [11]. This may give an answer to the problem of Prof. A. Zygmund.

Using this as a main tool, we may extend the Hilbert transform to the other direction. These are treated in chapter 2.

In chapter 3 we shall prove the reciprocal formula of this operator by the complex variable methods. Setting this result as the base of arguments, we treat analytic functions in a half-plane. The ordinary case is due to R.E.A.C. Paley-N. Wiener [25] and E. Hille-J.D. Tamarkin [15, 16].

*) This contain the detailed argument of papers published in the Proc. Japan Academy, vol. 34~5 (1958~9).