ON CONJUGATELY SIMILAR TRANSFORMATIONS

By

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Introduction. H. Nakano in this book [1] defined the *modulared semi-ordered linear space* $R(m)$, that is, R is a universally continuous[1)] semi-ordered linear space where a functional $m(a)$ $(a \in R)$ is defined such as the following seven properties are satisfied:

1) $0 \leqq m(a) \leqq +\infty$ for all $a \in R$;
2) if $m(\xi a)=0$ for all $\xi \geqq 0$, then $a=0$;
3) for any $a \in R$ there exists $\alpha > 0$ such that $m(\alpha a) < +\infty$;

(M) 4) for any $a \in R$, $m(\xi a)$ is a convex function of ξ;

5) $|a| \leqq |b|$ implies $m(a) \leqq m(b)$;
6) $a \frown b=0$ implies $m(a+b)=m(a)+m(b)$;
7) $0 \leqq a_\lambda \uparrow_{\lambda \in \Lambda} a$[2)] implies $\sup_{\lambda \in \Lambda} m(a_\lambda)=m(a)$.

This functional $m(a)$ $(a \in R)$ is called a *modular* on R. The well-known space $L_p([0,1])$ $(p \geqq 1)$ is one of examples of the modulared semi-ordered linear space, putting $m_p(a)=\int_0^1 \frac{1}{p} |a(t)|^p \, dt$ $(p \geqq 1)$.

Let R be a universally continuous semi-ordered linear space and $\bar{R}$ be the *conjugate space* of R, that is, the space of all universally continuous[3)] linear functionals on R. Especially when R is a modulared semi-ordered linear space by modular $m(a)$ $(a \in R)$, a functional $\bar{a} \in \bar{R}$ is said to be *modular bounded* if $\sup_{m(a) \leq 1} |(a, \bar{a})| < +\infty$. The space of all modular bounded functionals $\bar{R}^m$ is a universally continuous semi-ordered linear space. When we put for $\bar{a} \in \bar{R}$

$$\bar{m}(\bar{a})=\sup_{a \in R} \{(a, \bar{a})-m(a)\} \quad (\bar{a} \in \bar{R}), \tag{1}$$

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1) A semi-ordered linear space R is said to be universally continuous if for any system $a_\lambda \geqq 0$ $(\lambda \in \Lambda)$ there exists an element $\bigcap_{\lambda \in \Lambda} a_\lambda$ in R ([1], p. 17).

2) For any $\lambda_1, \lambda_2 \in \Lambda$ there exists $\lambda_3 \in \Lambda$ such that $a_{\lambda_1} \smile a_{\lambda_2} \leqq a_{\lambda_3}$ and $\bigcup_{\lambda \in \Lambda} a_\lambda = a$.

3) A linear functional $\bar{a}$, $(a, \bar{a})$ $(a \in R)$, is said to be universally continuous, if for any $a_\lambda \downarrow_{\lambda \in \Lambda} 0$ we have $\inf_{\lambda \in \Lambda} |(a_\lambda, \bar{a})|=0$ ([1], p. 81).