ON SEMI-LOWER BOUNDED MODULARS

By

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W. Orlicz and Z. Birnbaum proved in [7] that an Orlicz space $L_\Phi(G)$ is finite if and only if the function Φ satisfies the following condition: for some $\gamma>0$ and $t_0>0$, $\Phi(2t)\leqq\gamma\Phi(t)$ for every $t\geqq t_0$. (In case of mes $(G)=+\infty$, $\Phi(2t)\leqq\gamma\Phi(t)$ for all $t\geqq 0$.)

This fact was generalized for arbitrary monotone complete modulars on non-atomic space by I. Amemiya in [1], that is, suppose that R is a universally continuous semi-ordered linear space and has no atomic element, then every monotone complete finite modular on R is semi-upper bounded. T. Shimogaki showed in [8] a new simple proof of this Amemiya's Theorem. In this paper we investigate the properties of the conjugate modular of a semi-upper bounded modular, i.e. the semi-lower bounded modular. Throughout this paper we use the terminologies and notations used in [5].

In §1 we give corollaries of Amemiya's Theorem and a theorem relate to Amemiya's Theorem. In §2 we investigate the relations between a modular or the modular norms and semi-lower bounded modular. In §3 we express the properties of a semi-upper and semi-lower bounded modular.

§1. Let R be a universally continuous semi-ordered linear space and m be a modular on R[1]. A modular m is said to be "*finite*", if $m(x)<+\infty$ for every $x\in R$. A modular m is said to be "*monotone complete*", if for $0\leqq a_\lambda\uparrow_{\lambda\in\Lambda}$, $\sup_{\lambda\in\Lambda} m(a_\lambda)<+\infty$ there exists $a\in R$ for which $a_\lambda\uparrow_{\lambda\in\Lambda}a$. And a modular m is said to be "*semi-upper bounded*", if for every $\varepsilon>0$ there exists $\gamma=\gamma(\varepsilon)>0$ such that $m(x)\geqq\varepsilon$ implies $m(2x)\leqq\gamma m(x)$.

In [1] I. Amemiya proved:

Theorem 1.1. *Suppose that R has no atomic element, then every monotone complete, finite modular on R is semi-upper bounded.*

We say a modular m on R to be "*domestic*", if for any $a\in\{a:m(a)<+\infty, a\in R\}$ there exists $\xi=\xi(a)>1$ such that $m(\xi a)<+\infty$. On R, we define the two functionals $\|a\|$, $|||a|||$ $(a\in R)$ as follows:

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1) For the definition of the modular see H. Nakano [5].