SOME STUDIES ON PROJECTIVE FROBENIUS EXTENSIONS[1)]

By

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Introduction. Let A be a Frobenius algebra over a field K, that is, an algebra such that its first and second regular representations are equivalent. Then there exist bases $\{l_i\}$, $\{r_i\}$ of A over K such that

$$l_i a = \sum_{j=1}^{n} \lambda_{ij}(a) l_j ,$$

$$a r_i = \sum_{j=1}^{n} r_j \lambda_{ji}(a) \qquad (\lambda_{ij}(a) \in K , \quad i = 1, 2, \cdots, n)$$

hold for every element a in A. Let h_i's and k_i's be the K-linear transformations of A into K defined by

$$h_i(a) = \lambda_i$$

$$k_i(a) = \lambda'_i \qquad (i = 1, 2, \cdots, n) ,$$

where $a = \sum_{j=1}^{n} \lambda_j l_j = \sum_{j=1}^{n} r_j \lambda'_j$, λ_j, $\lambda'_j \in K$. Then for every element a in A we have

$$(1) \qquad a = \sum_{j=1}^{n} h_j(a) l_j = \sum_{j=1}^{n} r_j k_j(a) ,$$

$$(2) \qquad h_i(l_j a) = k_j(a r_i) \qquad (1 \leqq i, j \leqq n) .$$

These properties do not require that K is a field, and suggest a generalization of the notion of a Frobenius algebra.

Let Γ be a ring with a unit element 1 and Λ be a subring of Γ containing 1. Then we consider the following condition (A) which corresponds to (1) and (2):

(A) There exist element $\{l_i\}$, $\{r_i\}$ in Γ and Λ-linear mappings $\{h_i\} \subseteqq \mathrm{Hom}({}_{\Lambda}\Gamma, {}_{\Lambda}\Gamma)$ and $\{k_i\} \subseteqq \mathrm{Hom}(\Gamma_{\Lambda}, \Gamma_{\Lambda})$ such that both (1) and (2) hold for every element a in Γ.

On the other hand, in [6] Kasch defined that the ring extension Γ/Λ is

1) This is a doctoral dissertation at Hokkaido University.