ON THE NORMAL BASIS THEOREMS AND THE EXTENSION DIMENSION

By

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Recently, in his paper [7] one of the authors has presented several generalized normal basis theorems for a division ring extension, which contain as special cases the normal basis theorems given in [1] by Kasch (provided for division ring extensions). One of the purposes of this paper is to extend his results to simple rings. In §1, we shall prove those extensions, and add a decision condition for a normal basis element in a strictly Galois extension of a division ring, which is well-known in commutative case. Next, in §2, we shall treat exclusively an F-group of order p^e in a simple ring, and consider the relations between the extension dimension over the fixed subring and the order of the F-group. The principal theorem of §2 is an improvement of the result stated in [8] for a DF-group. As to notations and terminologies used in this paper, we follow [3] and [5].

§1. The following lemma has been given in [7][1], and will play a fundamental role in our present study.

Lemma 1. *Let $T\ni 1$ be a ring with minimum condition for right ideals, and let M, N be unital right T-modules.*

(i) *M is T-projective if and only if it is T-isomorphic to a direct sum of submodules each of which is T-isomorphic to a directly indecomposable direct summand of T.*

(ii) *If $M^{(m)}\simeq T^{(\omega)}$ for a positive integer m and an infinite cardinal number ω, then $M\simeq T^{(\omega)}$.*

(iii) *If $M^{(m)}\simeq T^{(t)}$ for positive integers m, t and $t=mq+r$ $(0\leq r<m)$, then $M\simeq T^{(q)}\oplus M_0$, where M_0 is a T-homomorphic image of T such that $M_0^{(m)}\simeq T^{(r)}$. In particular, if $m=t$ then $M\simeq T$.*

(iv) *If M is T-projective and $M^{(m)}\sim N^{(n)}$ with $m\leq n$ then $M\sim N$.*

Theorem 1. *Let $\mathfrak{H}$ be an N-group with $B=J(\mathfrak{H}, A)$, and $N\ni 1$ an $\mathfrak{H}$-invariant subring of A with minimum condition for right ideals such that A possesses a finite (linearly independent) right N-basis $\{x_1, \cdots, x_t\}$. If*

1) Numbers in brackets refer to the references cited at the end of this paper.