ON THE REPRESENTATION OF LARGE EVEN INTEGERS AS SUMS OF TWO ALMOST PRIMES. I

By

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The classical Goldbach problem, which still survives unsolved, is to prove that every even integer $\geqq 6$ is a sum of two prime numbers. In 1948 A. Rényi [4] succeeded, by making use of his refinement of the large sieve of Yu. V. Linnik, in proving that every even integer $\geqq 6$ is a sum of a prime and of an almost prime. Here an almost prime is a positive integer (>1) the total number of prime factors of which is bounded by a certain constant. Recently this result was sharpened in part by Ch.-D. Pan [3], who showed that every sufficiently large even integer can be represented as a sum of a prime and of an almost prime possessing at most five prime factors.

On the other hand, A. A. Buhštab [1] has proved that every large even integer can be written as a sum of two almost primes, each of which is composed of at most four prime factors. The purpose of the present paper is to improve this result of Buhštab. Indeed, we shall prove the following

Theorem. *Every sufficiently large even integer is representable as a sum of two integers, each of which has not more than three prime factors.*

We know that this result is originally due to A. I. Vinogradov [7]. However, as has been reviewed by H. Davenport [2], the exposition of Vinogradov in [7] does not seem to be quite clear. Thus it will be worth while, we believe, to give another proof for the theorem. Our proof of the above theorem is based on a combination of the sieve methods of Viggo Brun and of A. Selberg: in fact, it is substantially a deduction from an intermediate result obtained by Buhštab [1].

It should be noted that the following result can also be proved by the same argument *mutatis mutandis*: for every fixed integral value of $k \geqq 1$ there are infinitely many pairs of integers m, $m+2k$ with $V(m) \leqq 3$, $V(m+2k) \leqq 3$, where $V(n)$ denotes the total number of prime factors of n.

1. Throughout in this paper the letters d, k, m, n, r are used to denote positive integers, p, q to denote prime numbers, and x to denote a positive